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Math 117 - FALL 2024 - Common Final Exam

Print name:____

Section number:__________ **Instructor's name:**____________________________

Directions:

- This exam has 14 questions. Please check that your exam is complete, but otherwise keep this page closed until the start of the exam is called.
- Fill in your name, and your instructor's name.
- It will be graded out of 100 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- A formula sheet has been provided with this exam. You may not refer to any other notes during the exam.
- You may use a calculator which does not allow internet access. The use of any notes or electronic devices other than a calculator is prohibited.
- **Unless otherwise stated, round any constants to three decimal places if necessary.**

Good luck!

Question:	1	2	3	4	5	6	7	8
Points:	5	6	4	10	12	4	8	9
Score:								

Question:	9	10	11	12	13	14		Total
Points:	6	6	9	6	6	9		100
Score:								

1. (5 points) Let f be an increasing function defined for $x \geq 0$. The table gives values of $f(x)$ at selected values of x.

x	1	2	3	4	5
$f(x)$	-4	6	10	23	40

The function g is given by

$$g(x) = \frac{x^3 - 7}{x + 2}.$$

(a) The function h is defined by $h(x) = g(f(x))$. Find the value of $h(2)$ as a decimal approximation, or indicate that it is not defined.

$$h(2) = g(\underbrace{f(2)}_{6}) = g(6) = \frac{6^3 - 7}{6 + 2} = \frac{209}{8} = 26.125$$

(3 points)

(b) Find the value of $f^{-1}(-4)$, or indicate that is not defined.

$$f^{-1}(-4) = 1$$

(2 points)

2. (6 points) Find a possible formula for the polynomial function shown below.

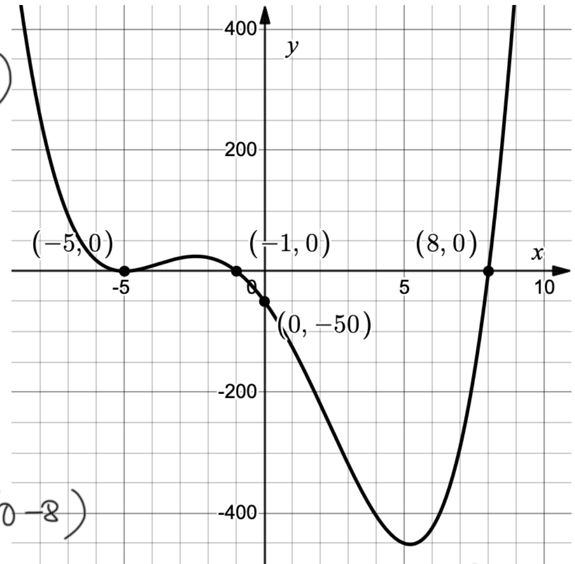
4th degree (1 point)

$y = a \cdot (x+5)^2 (x+1)(x-8)$

(2 points)

Using $(0, -50)$ to find a

(2 points)



$-50 = a \cdot (0+5)^2 \cdot (0+1) \cdot (0-8)$

$-50 = a \cdot (5)^2 \cdot (1) \cdot (-8)$

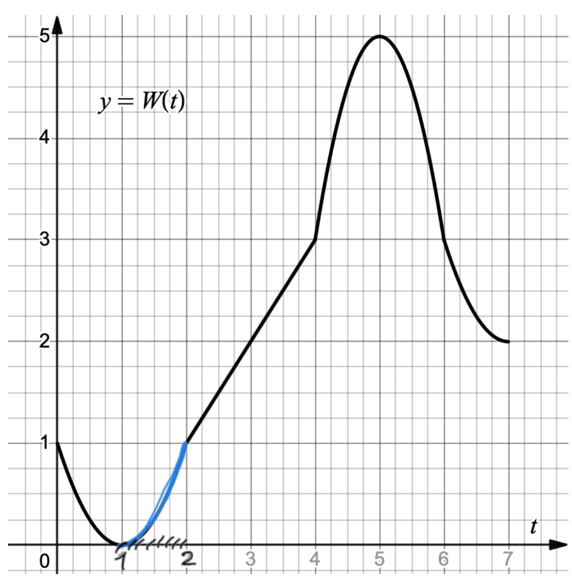
$a = \frac{1}{4}$

final formula

$y = \frac{1}{4} \cdot (x+5)^2 \cdot (x+1) \cdot (x-8)$

(1 point)

3. (4 points) The graph of $y = W(t)$ is shown for $0 \leq t \leq 7$. What are all intervals of t on which the function is both increasing and concave up? If none, write DNE.



$1 \leq t \leq 2$

4. (10 points) Let $k(x)$ be the function given by

$$k(x) = \frac{x^2 - 2x - 35}{x^3 - 3x^2 - 18x}. = \frac{(x+5)\cdot(x-7)}{x\cdot(x+3)\cdot(x-6)}$$

(a) Find the zeros of $k(x)$.

$x^2 - 2x - 35 = 0$

$(x+5)(x-7) = 0$ (or quadratic formula)

(2 points)

$x = -5, \quad x = 7$

(b) Find all vertical asymptotes.

(3 points)

$x^3 - 3x^2 - 18x = 0$

$x(x+3)(x-6) = 0$

$x = 0, \quad x = -3, \quad x = 6$

(c) What is the domain of $k(x)$?

$x \neq 0, \quad x \neq -3, \quad x \neq 6$

(2 points)

(d) Find a horizontal asymptote and describe the long-run behavior of the function $k(x)$ as $x \to +\infty$ and $x \to -\infty$.

(3 points)

$$k(x) \approx \frac{x^2}{x^3} = \frac{1}{x} \to 0$$

as $x \to \pm\infty$

horizontal asymptote $y = 0$

5. (12 points) A theater manager graphed weekly profits as a function of the number of patrons and found that the relationship was linear. One week the profit was \$9500 when 1300 patrons attended. Another week 1500 patrons produced a profit of \$11,400.

(3) (a) Find a formula for weekly profit, y, as a function of the number of patrons, x.

$$y = m \cdot x + b \quad , \quad m = \frac{11400 - 9500}{1500 - 1300} = \frac{1900}{200} = 9.5$$

$$b = 9500 - 9.5 \cdot 1300 = -2850 \qquad \boxed{y = 9.5 \cdot x - 2850}$$

(3) (b) Interpret the slope and the y-intercept.

$m = 9.5 \frac{\$}{\text{patron}}$

$b = -2850$ negative profit when zero patrons

(3) (c) If the weekly profit was \$20,900, how many patrons attended the theater?

$$20\,900 = 9.5 \cdot x - 2850 \Rightarrow 20900 + 2850 = 9.5x$$

$$23750 = 9.5 \cdot x \Rightarrow \frac{23750}{9.5} = x = 2500 \text{ patrons}$$

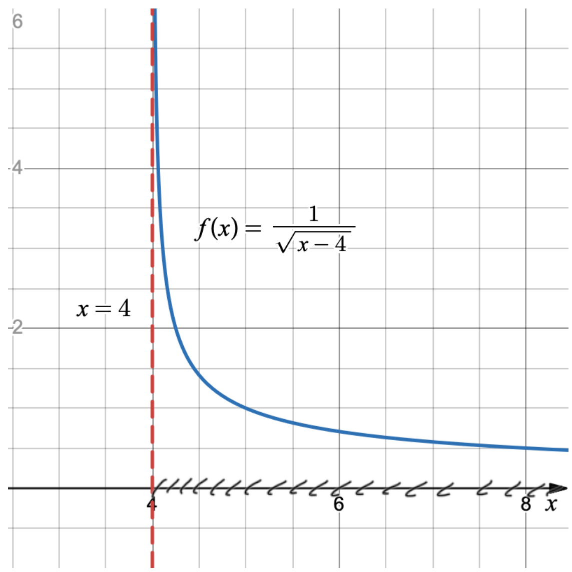
(3) (d) Find a formula for the number of patrons as a function of profit.

$$x = \frac{y + 2850}{9.5}$$

6. (4 points) Find the domain of the function

$$f(x) = \frac{1}{\sqrt{x-4}}$$

by examining its formula and/or its graph.



$x - 4 > 0$

$x > 4$

7. (8 points) Let

$$f(x) = \frac{5x-2}{1-3x}.$$

For what value of x is $f(x) = 2$?

(2) $\frac{5x-2}{1-3x} = 2$

(4)
$5x - 2 = 2\cdot(1-3x)$
$5x - 2 = 2 - 6x$
$5x + 6x = 2 + 2$

(2)
$11x = 4$
$x = \frac{4}{11}$

8. (9 points) A water fountain shoots a stream of water from ground level into the air forming a parabolic arc. The height $H(t)$, in feet, of the water stream at time t, in seconds, is modeled by the equation

$$H(t) = -16t^2 + 48t \quad = a\cdot(x-h)^2+k$$

$$a=-16,\ b=48,\ c=0$$

(3) (a) Find the time it takes for the water to reach its maximum height.

$$h = -\frac{b}{2a} = -\frac{48}{2\cdot(-16)} = \frac{3}{2}$$

(3) (b) What is the maximum height of the water?

$$k = H\left(\frac{3}{2}\right) = -16\cdot\left(\frac{3}{2}\right)^2 + 48\cdot\left(\frac{3}{2}\right) = -36+72 = 36$$

(3) (c) Find the time it takes for the water stream to come back to ground level.

$$-16t^2+48t = 0$$

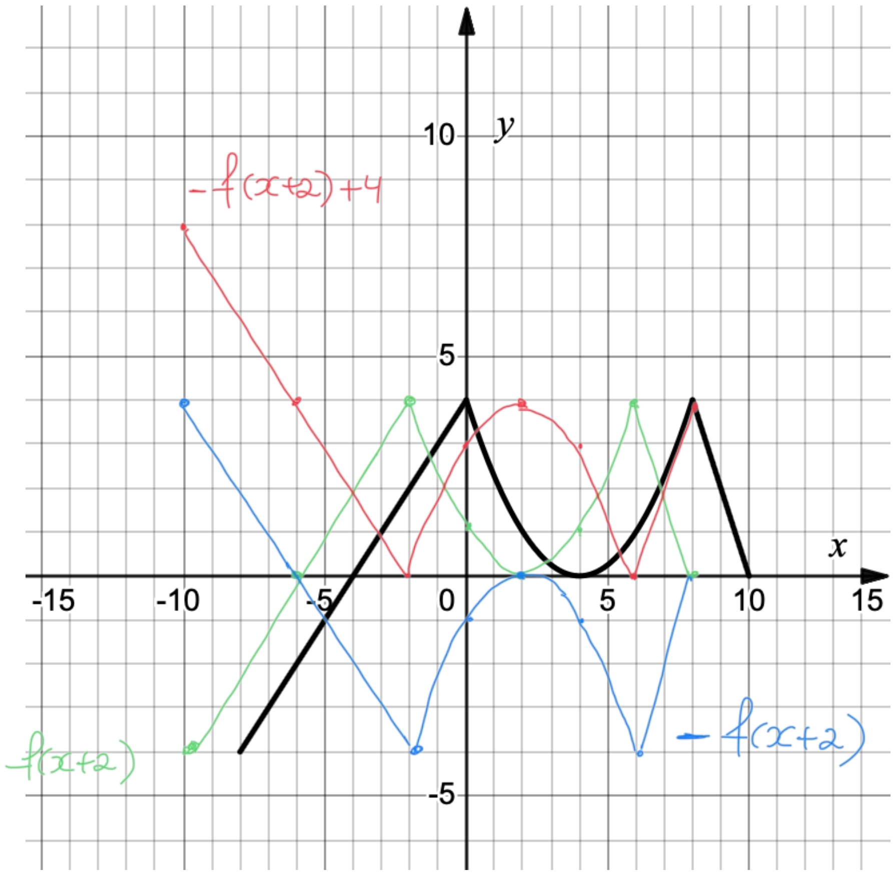
$$-16t\cdot(t-3) = 0$$

$$t=0,\quad t=3$$

9. (6 points) The graph of the function $f(x)$ is given below. Graph the following transformation of the function $f(x)$ on the same axes.

$$y = -f(x+2) + 4$$

2 points for each correct transformation



10. (6 points) The point $(2, -3)$ is on the graph of $y = p(x)$. Give the coordinates of the corresponding point on the graph of the transformation

$$q(x) = -3 \cdot p(2x) + 1.$$

$p(x)$: $(2,-3)$

$p(2x)$: $(1,-3)$

$-3 \cdot p(2x)$: $(1, 9)$

$-3 \cdot p(2x) + 1$: $(1, 10)$

2 points for each correct transformation

11. (9 points) At the Fun Slice Pizzeria it costs 2 cents per square inch to add pepperoni to a pizza.

(a) Find a formula for the cost $C(A)$ to cover A square inches of a pizza.

(3) $C(A) = A \cdot 0.02$

(b) The area of a circular pizza (in square inches) is a function of the radius r (in inches) is given by

$$A(r) = \pi r^2.$$

Find an expression for $C(A(r))$ and explain what it means in practical terms.

(3) $C(A(r)) = \pi \cdot r^2 \cdot 0.02$

(c) Find the cost in dollars of completely covering a circular pizza of radius 8 inches. Round your answer to the nearest penny.

(3) $C(A(8)) = \pi \cdot 8^2 \cdot 0.02 = \$4.02$

12. (6 points) In a microwave oven, cooking time t is inversely proportional to the square root of the amount of power used w. It takes 8 minutes to heat a frozen dinner at 625 watts.

$$t = \frac{K}{\sqrt{w}}$$

(3) (a) Write a formula for the cooking time, t, as a function of power level, w.

$$8 = \frac{K}{\sqrt{625}} \Rightarrow 8 \cdot 25 = K \Rightarrow t = \frac{200}{\sqrt{w}}$$

$$200 = K$$

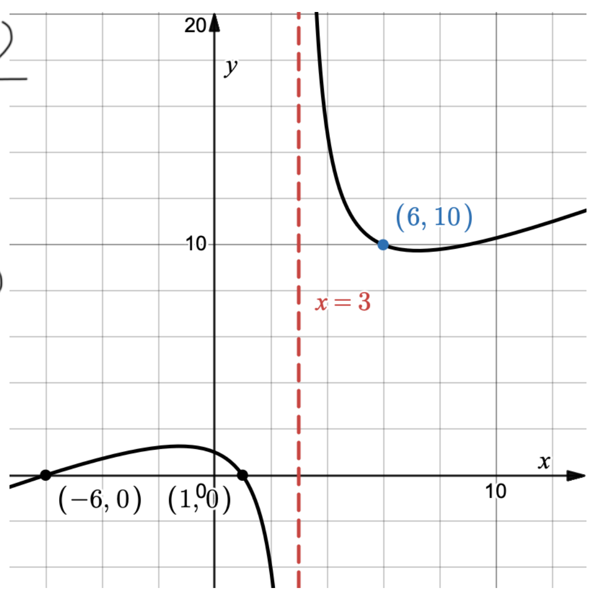
(3) (b) Find the cooking time needed to heat the frozen dinner at the power level $w = 324$ watts.

$$t = \frac{200}{\sqrt{324}} = \frac{200}{18} = 11.11 \text{ minutes}$$

13. (6 points) The function f is a rational function. Its graph is shown below. Give a possible formula for $f(x)$.

(2 points)

$$f(x) = a \frac{(x+6)\cdot(x-1)}{x-3}$$



(2 points)

Using the point (6,10)

$$10 = a \frac{(6+6)\cdot(6-1)}{(6-3)}$$

(1 point)

Calculations

$$10 = a \frac{12 \cdot 5}{3} \Rightarrow$$

$$a = \frac{1}{2}$$

(1 point) final formula

$$f(x) = \frac{1}{2} \frac{(x+6)\cdot(x-1)}{x-3}$$

14. (9 points) A floor-refinishing company charges \$2.5 per square foot to strip and refinish a tile floor for up to 500 square feet. Since the tiles generate toxic waste, there is an additional charge of \$350 for toxic waste disposal for any job that includes more than 200 square feet of tile.

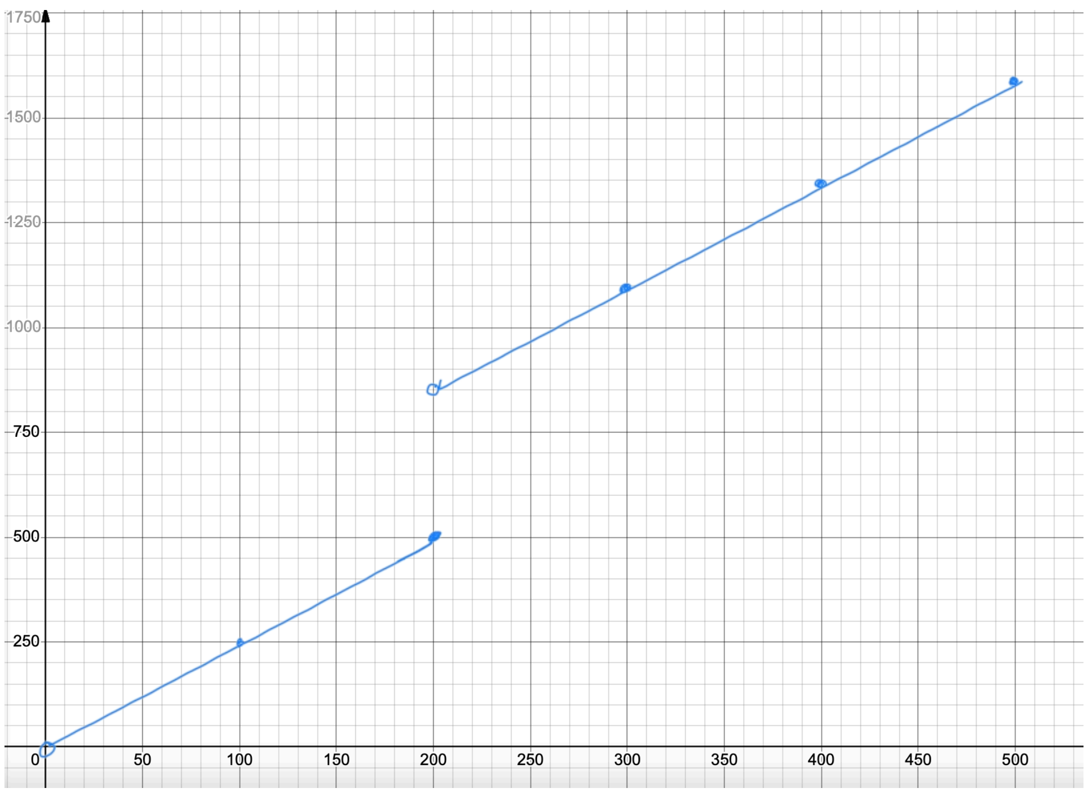
(a) Express the cost, y, of refinishing a floor as a piecewise-defined function of the number of square feet, x, to be refinished.

(3)

$$y = \begin{cases} 2.5 \cdot x, & 0 < x \le 200 \\ 2.5 \cdot x + 350, & 200 < x \le 500 \end{cases}$$

(b) Graph the function.

(3)



(3) (c) Give the domain and range.

Domain: $0 < x \le 500$

Range: $0 < y \le 500$,

$850 < y \le 1600$

Formulas

Average rate of change: $\dfrac{f(b) - f(a)}{b - a}$

Slope-intercept form: $y = b + mx$

Point-slope form: $y - y_0 = m(x - x_0)$

Standard form: $Ax + By = C$

Quadratic function: $y = ax^2 + bx + c$

Factored form: $y = a(x - r)(x - s)$

Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vertex form: $y = a(x - h)^2 + k$

Power function $y = kx^p$

Directly proportional: $y = kx$

Inversely proportional: $y = \dfrac{k}{x}$

Factored form of a polynomial: $p(x) = c(x - a_1)(x - a_2) \cdots (x - a_n)$